

Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS  
METHODS  
UNIT 1

**SOLUTIONS**

Section Two:  
Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 7 | 7 | 50 | 52 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 98 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has**twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 (8 marks)

(a) The variables and are linearly related so that when and when  
. State, with reasoning, whether the relationship between and is an example of direct proportion. (2 marks)

|  |
| --- |
| Solution |
| For direct proportion require that , a straight line through origin.  Since and then and so relationship is an example of direct proportion. |
| Specific behaviours |
| ✓ states relationship is direct proportion (with reasoning)  ü shows that or similar |

(b) Four points have coordinates and .

(i) If is the midpoint of and , determine the value of the constant and the value of the constant . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ value of  ü value of |

(ii) Determine the equation of the line that is perpendicular to and that passes through in the form , where and are integers and .

(4 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ slope of  ü slope of perpendicular line  ü correct equation of line, any form  ü correct equation of line in required form |

Question 9 (9 marks)

The time of sunrise, hours after midnight, on the day of the year in Esperance is closely modelled by

(a) Use the model to calculate, in hours and minutes, the time of sunrise on February.

(2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct value of  ü correct time in hours and minutes |

(b) Graph as a function of on the axes below for . (4 marks)

|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ endpoints  ü maximum  ü minimum  ü smooth curve |

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(c) Jill lives in Esperance and is woken by her alarm clock at am every day. How many days will she be awake before sunrise in a year of days? (3 marks)

|  |
| --- |
| Solution |
| when (see dotted line on graph).  when .  Hence awake days through and through , which is days. |
| Specific behaviours |
| ✓ indicates when am  ü indicates correct intervals of days  ü correct number of days |

Question 10 (8 marks)

Triangle has sides cm, cm and an area of cm2.

(a) Determine all possible values for the size of angle . (3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ substitutes into area formula correctly  ü solves for one correct angle  ü solves for both correct angles |

(b) Determine, showing use of trigonometry, the largest possible perimeter of triangle .

(3 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ü substitutes into cosine rule correctly  ü correct length of  ü correct perimeter |

(c) Determine, showing use of trigonometry, the size of angle when triangle has the largest possible perimeter. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ substitutes into sine rule correctly  ü solves for correct angle |

Question 11 (8 marks)

A calculator is used to randomly generate a whole number between and inclusive.

Event occurs when the number is a multiple of , event occurs when the number is odd and event occurs when the number is a square number.

(a) Determine the ordered set of all possible outcomes for the event

(i) . (2 marks)

|  |
| --- |
| Solution |
| Numbers that are in at least one of the sets: |
| Specific behaviours |
| ✓ indicates way to identify subset (Venn diagram, table, words, etc)  ü correct ordered list (*penalise lack of curly brackets once in question*) |

(ii) . (2 marks)

|  |
| --- |
| Solution |
| Numbers that are not in all three sets: |
| Specific behaviours |
| ✓ indicates way to identify subset  ü correct ordered list |

(b) Determine

(i) . (2 marks)

|  |
| --- |
| Solution |
| Multiples of three that are even: |
| Specific behaviours |
| ✓ identifies subset  ü correct probability |

(ii) . (2 marks)

|  |
| --- |
| Solution |
| Odd numbers that are neither square nor multiples of three: |
| Specific behaviours |
| ✓ identifies subset  ü correct probability |

Question 12 (7 marks)

In a random sample of cats, it was found that of the cats were males with green-coloured eyes, and of the females had green-coloured eyes.

(a) Determine the probability that a randomly selected cat from the sample

|  |
| --- |
| Solution |
| Let be event cat is female. |
| Specific behaviours |
| ✓ correct probability |

(i) is female. (1 mark)

(ii) is female or does not have green-coloured eyes. (2 marks)

|  |
| --- |
| Solution |
| Let be event cat has green-coloured eyes. |
| Specific behaviours |
| ✓ indicates appropriate method  ü correct probability |

(iii) is female given that it has green-coloured eyes. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates appropriate method  ü correct probability |

(b) Does the sample indicate possible independence of sex and eye colour for cats? Explain your answer. (2 marks)

|  |
| --- |
| Solution |
| Yes, the events may well be independent.  It can be seen from probabilities in part (a) that is very close to and as an indicator for possible independence, the closer the better. |
| Specific behaviours |
| ✓ states independent  ü explanation using conditional probability |

Question 13 (9 marks)

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</EFOFEX>The shaded regions shown in the diagram form  
a logo that is to be cut from a plastic sheet.  
  
The boundaries of the shaded regions are  
parts of triangle , where ,  
and arcs of two concentric circles with  
centre and radii of cm and cm.

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct conversion |

(a) Express exactly in radians. (1 mark)

(b) Determine the total length of the two curved and three straight cuts that must be made by the cutting machine, giving your answer to the nearest centimetre. (4 marks)

|  |
| --- |
| Solution |
| Length of arcs ():  Length of :  Total lengths of cuts: cm. |
| Specific behaviours |
| ✓ calculates one correct arc length  ü indicates correct use of cosine rule for  ü correct length of  ü correct total length |

(c) Determine the area of logo, giving your answer to the nearest square centimetre.

(4 marks)

|  |
| --- |
| Solution |
| Area of sector:  Area of segment:  Total area: cm2. |
| Specific behaviours |
| ✓ correct area of sector  ü indicates correct use of segment area formula  ü correct area of segment  ü adds segment and sector to obtain correct total area |

Question 14 (9 marks)

A function defined by , where and are constants, passes through the points and .

(a) Determine the value of and the value of . (3 marks)

|  |
| --- |
| Solution |
| Solve simultaneously using CAS: |
| Specific behaviours |
| ✓ uses points to form two equations  ü value of  ü value of |

(b) Draw the graph of on the axes below, clearly indicating the coordinates of all axes intercepts and equations of any asymptotes. (4 marks)

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|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ location of intercept  ü equations of both asymptotes  ü RHS, smooth curve, thru'  ü LHS, smooth curve, thru' |

(c) State the equations of all asymptotes of the graph of . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ equations of horizontal asymptote  ü equations of vertical asymptote |

Question 15 (7 marks)

Events and are such that and .

(a) When and are mutually exclusive, determine . (1 mark)

|  |
| --- |
| Solution |
| Mutually exclusive  Hence |
| Specific behaviours |
| ✓ correct probability |

(b) When , determine . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct method  ü correct probability |

(c) When , determine . (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct  ü correct probability |

(d) When and are independent, determine . (2 marks)

|  |
| --- |
| Solution |
| Independent |
| Specific behaviours |
| ✓ indicates correct  ü correct probability |

Question 16 (8 marks)

A group of students is needed to assist in the school office. The group is to be selected from a class of , in which are day students and the remainder boarders.

(a) Determine the number of different groups that can be selected. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct use of combinations  ü correct number |

(b) Determine the number of different groups that can be selected containing no day students. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates correct use of combinations  ü correct number |

(c) A group is randomly selected from the class. Determine the probability that the group

(i) contains only day students. (2 marks)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ indicates appropriate method to derive numerator  ü correct probability |

(ii) contains at least one boarder and at least one day student. (2 marks)

|  |
| --- |
| Solution |
| Number of groups with only boarders or only day students is .  All other groups will have at least one of each, which is .  Hence |
| Specific behaviours |
| ✓ indicates appropriate method to derive numerator  ü correct probability |

Question 17 (8 marks)

(a) The graph of the quadratic function has roots at and and the range of is . Use an algebraic method to determine . (4 marks)

|  |
| --- |
| Solution |
| Axis of symmetry is midway between roots and so turning point at .  Using a root, .  Hence . |
| Specific behaviours |
| ✓ uses range and symmetry to identify turning point  ü writes equation in turning point form with constant  ü evaluates constant  ü states |

(b) The area of square is cm2 more than six times the area of square , and the difference in the perimeters of the two squares is cm. Determine the least possible area of square , the smaller of the squares. (4 marks)

|  |
| --- |
| Solution |
| Side of must be cm longer than side of .  If side of is , then side of is .  Hence  Least area when : |
| Specific behaviours |
| ✓ indicates difference in the side lengths  ü forms quadratic equation  ü solves quadratic equation  ü states least area |

Question 18 (8 marks)

As part of a statistics project a student accessed data from a sample of people about their age and whether they were a smoker. of the people were aged under , and of this age group were smokers. Altogether, there were smokers in the sample.

(a) Use the above information to complete the relative frequency table below, rounding entries to three decimal places. (3 marks)

|  |
| --- |
| Solution |
| Non-smokers :  Smokers 3+:  Non-smokers 3+:  Frequency table:  Divide each frequency by to obtain relative frequency table. |
| Specific behaviours |
| ✓ indicates two correct frequencies  ü indicates all correct frequencies  ü correct relative frequencies |

|  |  |  |
| --- | --- | --- |
| Age | Tobacco Use | |
| Non-smoker | Smoker |
| Under |  |  |
| or older |  |  |

(b) A person is randomly selected from the sample. Determine the probability that

(i) they are a non-smoker. (1 mark)

|  |
| --- |
| Solution |
| OR |
| Specific behaviours |
| ✓ correct probability |

(ii) they are a smoker or aged or older. (1 mark)

|  |
| --- |
| Solution |
| OR |
| Specific behaviours |
| ✓ correct probability |

(c) If two selections are made at random from the sample (with replacement), determine the probability that one person is a smoker, and the other is not. (3 marks)

|  |
| --- |
| Solution |
| Let event be selection is smoker, so that .  Hence .  *NB Exact values:* |
| Specific behaviours |
| ✓ probability of  ü probability of  ü probability of |

Question 19 (9 marks)

The graph of the cubic polynomial passes through the points , and has a local maximum at .

(a) Use the above information to sketch the graph of on the axes below. (3 marks)

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|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ smooth cubic curve  ü locates local maximum  ü locates intercepts |

Let , where and are constants.

(b) Determine the value of each of the constants and . (3 marks)

|  |
| --- |
| Solution |
| Factored form of cubic is  Hence  And so |
| Specific behaviours |
| ✓ correct factored form of cubic  ü expands cubic  ü correct value for each constant |

(c) Another cubic polynomial is defined by . Determine the value(s) of the constant so that the graphs of and do not intersect. (3 marks)

|  |
| --- |
| Solution |
| For intersection require .  Hence . For no intersection, this quadratic must have no solution and so discriminant, , must be less than zero.  Hence and so . |
| Specific behaviours |
| ✓ equates cubic equations and obtains quadratic  ü uses discriminant to form inequality  ü correct range of values for |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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